In the absence of a magnetic field, Eckert[5] has shown that the $\frac{1}{7}$-power law for forced-convection turbulent flow can be applied to obtain expressions for u, θ, τ_w, and q in the free-convection case. Moffatt[6] applied the $\frac{1}{7}$-power law to obtain these expressions in forced-convection turbulent flow in the presence of a magnetic field. In accord with Eckert and Moffatt, let

$$u = u_e(y/\delta)^{1/7}[1 - (y/\delta)]^4 \tag{3}$$

$$\theta = \theta_w[1 - (y/\delta)^{1/7}] \tag{4}$$

$$\tau_w = 0.0225\,\rho u_e^2(\nu/u_e\delta)^{1/4} \tag{5}$$

$$q_w = 0.0225\,g\rho C_p u_e \theta_w(\nu/u_e\delta)^{1/4}Pr^{-2/3} \tag{6}$$

$$u_e = C_1 x^m \tag{7}$$

$$\delta = C_2 x^n \tag{8}$$

in the momentum and the energy equations and obtain

$$0.0523C_1^2C_2(2m + n)x^{2m+n-1} = 0.125g\beta\theta_wC_2x^n - 0.130(\sigma B_0^2/\rho)C_2C_1x^{n+m} - 0.0225\nu^{1/4}C_1^{7/4}C_2^{-1/4}x^{2m-1/4(m+n)} \tag{9}$$

$$0.0366C_2C_1(n + m)x^{n+m-1} = 0.0225C_1^{3/4}C_2^{-1/4}\nu^{1/4}\rho^{-2/3}x^{m-(1/4)(m+n)} \tag{10}$$

By imposing the condition that B_0 must vary as $x^{-1/4}$, it is possible to solve for m and n by equating the exponents of x. Performing this operation, one gets $m = \frac{1}{2}$, $n = \frac{7}{10}$. With these values, Eqs. (9) and (10) now can be solved for C_1 and C_2, and, with the expressions obtained for C_1 and C_2, Eqs. (7) and (8) can be combined with Eq. (6) and arranged in dimensionless form as suggested by Cole:[7]

$$Nux = 0.002Pr^{1/3}Ra^{2/5}[1.27M + (1.62M^2 + 2.25Pr^{2/3} + 4.5)^{1/2}]^{-4/5} \tag{11}$$

Where Nu, Pr, Ra are the Nusselt, Prandtl, and Rayleigh numbers, respectively, and M is the magnetic parameter defined the same as in Ref. 3:

$$M = B_0^2x^{1/2}\sigma/\rho(g\beta\theta_w)^{1/2}$$

In the absence of a magnetic field, $M = 0$, and the reduction in heat transfer $(Nux)_w/(Nux)_0$ becomes

$$\frac{(Nux)_w}{(Nux)_0} = \left[\frac{(4.5 + 2.25Pr^{2/3})^{1/2}}{1.27M + (1.62M^2 + 4.5 + 2.25Pr^{2/3})^{1/2}}\right]^{4/5} \tag{12}$$

Figure 1 gives a comparison of the magnetic effect on the reduction in heat transfer between the laminar case[3] and the turbulent case derived here. It can be seen that for a given magnetic field much greater reduction in heat transfer is obtained in the turbulent case.

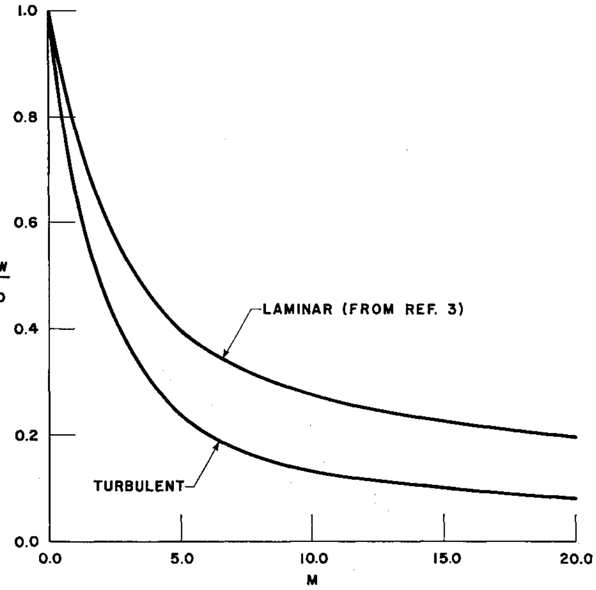

Fig. 1 Reduction in heat transfer $(Nux)w/(Nux)_0$ vs magnetic parameter M for laminar and turbulent flow ($Pr = 0.72$)

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Hypersonic Wake Transition

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RECENT data taken at Massachusetts Institute of Technology[1] have yielded estimates of the transition behavior for the near wakes of blunt bodies at high M_∞ and sharp bodies for $M_\infty<10$. Representative values from these data, plotted as transition distance from the estimated wake neck position vs freestream Reynolds number, are shown in Fig. 1. The solid line in the figure was taken directly from Ref. 1. In Fig. 2, the same data are plotted vs local Reynolds number, $R_{f,d} = \rho_fU_fd/\mu_f$. For the sphere data, the fluid properties were evaluated both by using computed values at the axis for an inviscid flow and also by using values at the edge of the turbulent wake (computed via the method of Ref. 2). The solid line is the authors' best estimate of the true laminar values. Several interesting features are exhibited by these and other unpublished data:

1) At high Reynolds numbers the transition distance for the cone seems to "stick" at a fixed distance from the body, whereas for the sphere the transition location continues to move toward the wake neck as the Reynolds number is increased.

2) Transition distance appears to vary linearly with $R_{f,d}$ for both the sphere and the cone in the region $x_{tr}/d < 30$,

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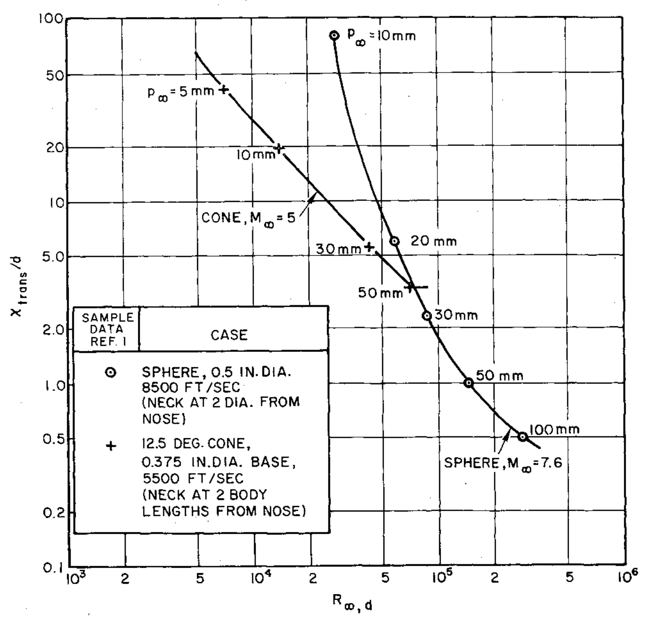

Fig. 1 Distance from wake neck to transition location vs freestream Reynolds number

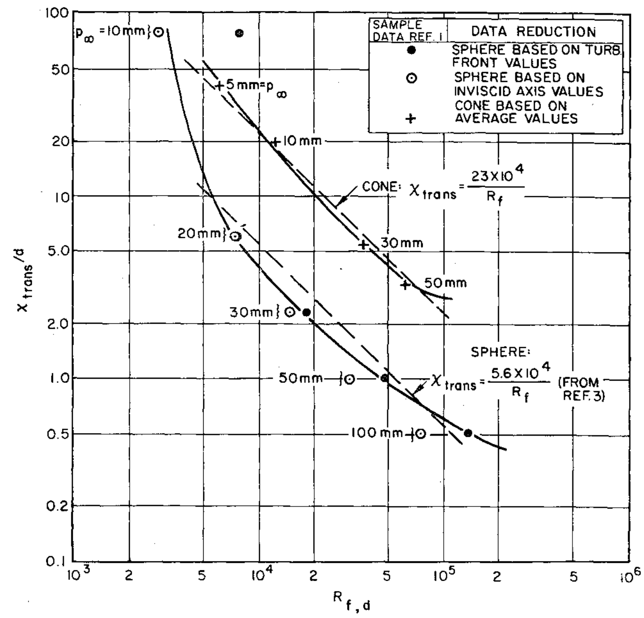

Fig. 2 Distance from wake neck to transition location vs local wake "edge" Reynolds number

indicating a constant $R_{f,x}$ for transition. This result is in good agreement with the data reported in Ref. 3 for blunt bodies, where it was found that, approximately, $R_{f,x_{tr}} = 5.6 \times 10^4$. However, Fig. 2 indicates that the sharp-body transition $R_{f,d}$ differs from that of the blunt body by a factor of about 4.

3) Far downstream ($x_{tr}/d > 50$) the transition location appears to increase sharply toward an asymptotic value as $R_{f,d}$ is lowered, at least for the blunt body. A "lower critical" wake Reynolds number, below which the wake is completely laminar, seems to be indicated.

4) Additional unpublished data taken at Massachusetts Institute of Technology at different flight speeds indicate general agreement with the results of Fig. 1; i.e., when transition distance is plotted vs $p_\infty d$, the data appear to be independent of flight speed. This result also has been reported in Ref. 4.

An attempt will be made to explain, at least qualitatively, the results 1, 3, and 4 just discussed. A complete explanation of result 2 is not available at the moment. However, the finite amplification rate for disturbances in the wake should be a relevant factor. It will be assumed in the following arguments that $M_\infty \gg 1$, that the flow is chemically frozen with $\gamma = \frac{7}{5}$, and that the Reynolds numbers are sufficiently low so that the flow upstream of the wake neck remains laminar.

Sticking of Transition for Sharp Bodies

Suppose that a disturbance from a fluid particle external to the wake is impressed at the "edge" of the wake core. The signal cannot reinforce itself at the wake axis if the following approximate relation holds:

$$(U_f - U_0) - a_f > a_0 \tag{1}$$

where subscript f denotes quantities at the edge of the wake, 0 denotes quantities at the wake axis, and U and a are the fluid velocity and local speed of sound, respectively. Alternatively, (1) may be written as

$$M_f - 1 > (1/a_f)(U_0 + a_0) \tag{2}$$

At the neck, where the dividing streamline (which divides the recirculating base flow from the external flow) crosses the axis, (2) becomes

$$M_f - 1 > (h_0/h_f)^{1/2} \tag{3}$$

The enthalpy ratio is approximately

$$\frac{h_0}{h_f} \approx \frac{\alpha(1 + M_\infty^2/5)}{1 + M_\infty^2/5\ (1 - \beta_1^2)} \tag{4}$$

where $\beta_1 = U_f/U_\infty$ and has typical values of 0.75 to 0.85 for blunt bodies and 0.99 for sharp bodies. The quantity $\alpha = h_0/H_\infty$, where H_∞ is the stagnation enthalpy. Since $M_f \approx M_\infty$ for sharp bodies and $M_f \approx 2$ to 3 for blunt bodies, then (3) becomes approximately

$$1.5 > (3\alpha)^{1/2}\ \text{(blunt)} \qquad 1 > (\alpha/5)^{1/2}\ \text{(sharp)}$$

Estimates[2] of the value of α indicate that it is about $\frac{1}{2}$, thus indicating that transition for a blunt body may be close to the neck, whereas the flow near the neck of sharp bodies is stabilized. Away from the neck where the axis velocity has increased to, say, βU_f, then (2) becomes

$$M_f\ (1 - \beta) - 1 > (h_0/h_f)^{1/2} \tag{5}$$

To determine the location at which this condition is satisfied, a solution of the laminar near-wake for non-negligible values of $1 - \beta$ is required. For a blunt body, (5) is satisfied for $\beta \approx 0.1$, and, since the axis velocity increases rapidly for small x ($U_f - U_0$ varies approximately as $1/x$), very short distances from the neck are implied. On the other hand, since the external Mach number for sharp bodies is considerably higher than for blunt bodies, an increased stabilization effect is to be expected. For example, for a 12.5° (half-angle) cone, the external Mach number even for $M_\infty = 6$ is about 5. Thus, with $\alpha = \frac{1}{2}$, a value of $\beta \approx 0.5$ must be reached before the external signals reinforce. For $M_\infty = 20$, then, β must be as large as 0.6. The conclusion is that the transition point becomes "stuck" at a much greater distance from the neck for sharp bodies.

Lower Critical Wake Reynolds Numbers

Far downstream from the body, one intuitively expects transition to occur only as long as a suitable Reynolds number is above some lower critical value. One possibility for this "wake Reynolds number" is $R_W = \bar{\rho}(U_f - U_0)\ b(x)/\bar{\mu}$, where $\bar{\rho}$ and $\bar{\mu}$ are appropriately averaged values over the wake width $b(x)$. Assuming that $p \sim \rho h$ is constant across the wake and that $\mu \sim T^{1/2}$, $h_0/h_f > 1$, then, with $\bar{\rho} = (\rho_f + \rho_0)/2$, $\bar{\mu} = (\mu_f + \mu_0)/2$, $\delta U = (U_f - U_0)/U_f$,

$$R_W = R_{f,d}(h_f/h_0)^{1/2}\,\delta U[b(x)/d] \qquad (6)$$

Using the results of linearized axisymmetric wake theory as given, for example, by Goldstein,[5]

$$\delta U \sim \frac{C_{Df}R_{f,d}}{x/d} \qquad \frac{b(x)}{d} \sim \left(\frac{x/d}{R_{f,d}}\right)^{1/2} \qquad (7)‡$$

where C_{Df} is the wake drag coefficient. Applying the Crocco integral ($P_r = 1$, $dp/dx \approx 0$)

$$\frac{h_0}{h_f} \approx 1 + \frac{(2\,\beta_1^2 - 1 + \alpha)\,M_\infty^2/5}{1 + M_\infty^2/5\,(1 - \beta_1^2)}\,\delta U \qquad (8)$$

for $\delta U < 1$ when $M_\infty^2/5$ is large. Thus

$$h_0/h_f \approx 0.3\,M_\infty^2\,\delta U \text{ (sharp bodies)}$$

$$h_0/h_f \approx 3\,\delta U \text{ (blunt bodies)} \qquad (9)$$

Inserting (7) and (9) into (6) gives

$$R_W \sim \frac{1.8(C_{Df})^{1/2}}{M_\infty}\,R_{f,d} \text{ (sharp)}$$

$$R_W \sim 0.6(C_{Df})^{1/2}\,R_{f,d} \text{ (blunt)} \qquad (10)$$

From the analysis of Ref. 2, $C_{Df} \sim R_{\infty,d}^{-1/2}$; hence (10) indicates that, over the same approximate range of $R_{\infty,d}$, equal values of R_W are obtained for sharp and blunt bodies when

$$(3R_{f,d}/M_\infty)_{\text{sharp}} \approx (R_{f,d})_{\text{blunt}} \qquad (11)$$

Dependence on Flight Speed

If isentropic flow behind the normal bow shock of a blunt body is assumed, then for a given freestream temperature and a constant γ of $\frac{7}{5}$,

$$R_{f,d} \sim \frac{M_\infty^8[5 + M_\infty^2(1 - \beta_1^2)]^2}{(7\,M_\infty^2 - 1)^{5/2}\,(5 + M_\infty^2)^{7/2}}\,p_\infty d$$

Hence, for $M_\infty \gg 1$,

$$R_{f,d} \sim p_\infty d \text{ (blunt)} \qquad (12)$$

alone, and for this reason the flight speed is irrelevant so long as it is sufficiently high. Experimental data obtained thus far have been at flight speeds insufficient for this conclusion to be strictly valid; however, in these cases the dependence on flight speed should be weak. But for sharp bodies

$$R_{f,d} \sim M_\infty p_\infty d \text{ (sharp)} \qquad (13)$$

and the flight speed enters directly into this Reynolds number.

For the stability Reynolds number, inserting (12) or (13) into (10) gives

$$R_W \sim 1.8(C_{Df})^{1/2}\,p_\infty d \text{ (sharp)}$$

$$R_W \sim 0.6(C_{Df})^{1/2}\,p_\infty d \text{ (blunt)} \qquad (14)$$

indicating that, so long as $C_{Df}^{1/2} \sim (R_{\infty,d})^{-1/4}$ is about constant, stability of both sharp and blunt-body wakes depends on $p_\infty d$ alone; moreover, the lower critical value of $p_\infty d$ for blunt bodies may be somewhat larger than the value for sharp bodies.

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Solid Propellant Exhaust Simulation

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THE purpose of this paper is to record a procedure for simulating exhausts of solid propellant engines by means of liquid, gaseous, and slurry systems. The method was devised some years ago and since has been developed and demonstrated, originally at Aeronutronic[1] and later at several other installations. The concept has many obvious applications and a major cost advantage over testing exhaust effects by use of solid rocket grains.

A given solid propellant can, in principle, be duplicated identically by any of a large number of combinations of liquid and/or gaseous or slurry systems. The duplications can be achieved with regard to both chamber temperature and chemical composition.

The composition and thermodynamic properties of an exhaust stream in thermodynamic equilibrium are determined uniquely by the atomic composition, the temperature, and the pressure. The pressure is fixed by the engine design, and the temperature is governed by the enthalpy of formation of the propellant. Consequently, in order to simulate the exhaust of a given solid propellant engine with a liquid, gaseous, or slurry system, the important parameters to consider are atomic composition and enthalpy of formation. The procedure is best illustrated by means of an example.

Suppose it is required to simulate the exhaust from an aluminized solid propellant of composition $C_a\,H_b\,O_c\,N_d\,Cl_eAl_f$ and enthalpy of formation Hf/100 g. If one selects seven chemicals, which among them contain the desired elements, one can, by a simple mass- and heat-balance process, duplicate the composition and enthalpy of formation precisely. In general, $N + 1$ ingredients are needed where N is the number of elements in the solid propellant. In the case cited, the components of the mixture might be chosen as $Al(s)$, $C_2HCl_3(l)$ (trichloroethylene), $C_2H_8N_2\,(l)$ (unsymmetrical dimethylhydrazine), $N_2(g)$, $O_2(g)$, $CH_{1.942}(l)$ (JP-4), and $H_2O(l)$. The aluminum would be used as a slurry with one or more of the liquids. The following equations then are set up:

$$a = 2\,N_{C_2H_3Cl} + 2\,N_{C_2H_8N_2} + N_{CH_{1.942}}$$

$$b = N_{C_2HCl_3} + 8\,N_{C_2H_8N_2} + 1.942\,N_{CH_{1.942}} + 2\,N_{H_2O}$$

$$c = 2\,N_{O_2} + N_{H_2O}$$

$$d = 2\,N_{C_2H_8N_2} + 2N_{N_2}$$

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